PREDICTION OF RAIN ATTENUATION SERIES WITH DISCRETIZED SPECTRAL MODEL

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1. INTRODUCTION

The rain attenuation plays a major role in the design of satellite-to-earth links operating at high frequencies such as Ka band and Ku band. The signal suffers from a strong attenuation during the propagation path as raindrops absorb and scatter radio waves, which results in an increase of transmission error and reduction of the system availability [1]. Predicting rain attenuation is one of the vital steps to be considered when analyzing a satellite communication link. Adaptive code mechanism or adaptive power control can be used to increase the transmission efficiency and to reduce the transmission power, on condition that the attenuation is predicted in some way.

Several models of the rain attenuation were introduced in [2] : spectral (SPL) model, two-sample model (TSM), second-order Markov chain (2MC) model, N-states Markov chain model, ITALSAT data-based model and synthetic storm technique (SST). Among these models the spectral model has a good correspondence with known properties of rain and its simplicity allows simulations of communication link performance under the influence of rain attenuation. In [3], the conditional probability distribution of the predicted attenuation based on the analog model was given, which allows an optimal prediction in the statistical sense. However, simulators often operate with the discretized model during the analysis of communication system. Predictions based on the discretized model were also studied in literature based on the filter design method [4] or on the online prediction method [5]. In this paper, we derive the conditional distribution of the predicted attenuation and give the optimal estimation bound based on the discretized model. The results can also be used for other systems with the similar model.

2. PREDICTION BASED ON THE DISCRETIZED SPECTRAL MODEL

2.1. Discretized spectral model

Relying on [3, 6], this spectral model partly aims at synthesizing rain attenuation series as well as scintillation time series. Fig. 1 shows the relationship between a white Gaussian noise input $w(n)$ and the instantaneous rain attenuation $a(n)$ in dB synthesized by the discrete SPL model. The digital filter $H(z)$ is a first-order system characterized by

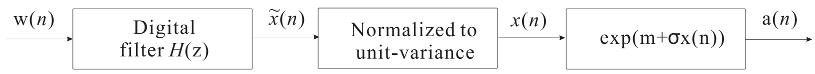

Fig. 1. discretized spectral model of rain attenuation

$$H(z) = \frac{a + az^{-1}}{1 - dz^{-1}}$$

where filter coefficients a and d depend on the parameters of prototype analog systems. The output $\tilde{x}(n)$ can be expressed by the difference equation

$$\tilde{x}(n) = aw(n) + aw(n-1) + dx(n-1) \tag{1}$$

with variance $\sigma_{\tilde{x}}^2 = \frac{2a^2}{1-d}$. The relation between the normalized variable $x(n)$ and the input process $w(n)$ is then written as

$$x(n) = \frac{1}{\sigma_{\tilde{x}}}(aw(n) + aw(n-1)) + dx(n-1) \tag{2}$$

Due to the centered input $w(n)$, the output of the difference equation $x(n)$ is also centered and has unit variance, i.e. $E[x(n)] = 0$ and $\sigma_x^2 = 1$. The final output $a(n)$ is obtained by passing $x(n)$ through the nonlinear module

$$a(n) = \exp(m + \sigma x(n)) \tag{3}$$

where m and σ are the parameters of this module.

2.2. The conditional pdf of $a(n+M)$

In this paper we shall determine the distribution of the attenuation at M instant later, conditioned by the observed values at instants $n, n-1, \cdots$, namely $P(a(n+M)|a(n), a(n-1), \cdots)$. The optimal predicted value can then be estimated by taking the expectation of this distribution. According to (1), the attenuation value to estimate at instant $n+M$ can be expressed by

$$\begin{aligned} a(n+M) &= \exp(m + \sigma x(n+M)) \\ &= \exp\left(m + \sigma(\frac{a}{\sigma_{\tilde{x}}}w(n+M) + \frac{a}{\sigma_{\tilde{x}}}w(n+M-1) + dx(n+M-1))\right) \end{aligned}$$

If $x(n+M-1)$ is expanded until the present instant n, the attenuation $a(n+m)$ can be rewritten as

$$a(n+M) = \exp\left(m + \frac{\sigma}{\sigma_{\tilde{x}}}aw(n+M) + \frac{\sigma}{\sigma_{\tilde{x}}}a\sum_{i=1}^{M-1} d^{i-1}(1+d)w(n+M-i) + \frac{\sigma}{\sigma_{\tilde{x}}}ad^{M-1}w(n) + \sigma d^M x(n)\right) \tag{4}$$

Considering the model (3), the intermediate variable $x(n)$ can be considered as known if $a(n)$ is known. Let $\tilde{\Gamma} = \frac{\sigma}{\sigma_{\tilde{x}}}aw(n+M) + \frac{\sigma}{\sigma_{\tilde{x}}}a\sum_{i=1}^{M-1} d^{i-1}(1+d)w(n+M-i)$. The expansion of $a(n+M)$ is expressed by the product of the following three terms

$$a(n+M) = \Gamma\,\nu_1\,\nu_2 \tag{5}$$

where

$$\Gamma = \exp(\tilde{\Gamma}) \tag{6}$$

$$\nu_1 = \exp(m + d^M(\log(a(n)) - m)) \tag{7}$$

$$\nu_2 = \exp\left(\frac{\sigma}{\sigma_{\tilde{x}}}ad^{M-1}w(n)\right) \tag{8}$$

First, it should be noticed that $\tilde{\Gamma}$ is a linear combination of white Gaussian variables $w(n+M), w(n+M-1), \ldots, w(n+1)$, hence $\tilde{\Gamma}$ follows a Gaussian distribution, with the mean $\mu_{\tilde{\Gamma}} = 0$ and the variance $\sigma_{\tilde{\Gamma}}^2 = \frac{\sigma^2 a^2}{\sigma_{\tilde{x}}^2}\left(1 + \sum_{i=1}^{M-1} d^{2i-2}(1+d)^2\right)$. With the exponential function $\exp(\cdot)$, the random variable Γ follows a log-normal distribution

$$P_\Gamma(\gamma) = \frac{1}{\sqrt{2\pi}\sigma_{\tilde{\Gamma}}\gamma}\exp\left(-\frac{(\log\gamma - \mu_{\tilde{\Gamma}})^2}{2\sigma_{\tilde{\Gamma}}^2}\right) \tag{9}$$

with the mean $\mu_\Gamma = \exp(\mu_{\tilde{\Gamma}} + \sigma_{\tilde{\Gamma}}^2/2)$ and the variance $\sigma_\Gamma^2 = \exp(2\mu_{\tilde{\Gamma}} + 2\sigma_{\tilde{\Gamma}}^2) - \exp(2\mu_{\tilde{\Gamma}} + \sigma_{\tilde{\Gamma}}^2))$.

Secondly, at any instant n, all components in the expression of ν_1, including $a(n)$, are known variables. Thus this item is deterministic and it only has influence on the amplitude of $a(n+M)$.

In the item ν_2, the white noise process $w(n)$ is not directly known. However, as $x(n)$ is generated by stimulating $H(z)$ with $w(n)$, the value of $w(n)$ can be estimated by passing it through the inverse system $H^{-1}(z) = \frac{1-dz^{-1}}{a+az^{-1}}$. The unique pole $z=-1$ on the unit circle indicates that this inverse system is stable but sensitive to the initial condition. Therefore a relaxation parameter η very close to 1 is introduced to improve the stability of the inverse filter. We then have

$$\hat{w}(n) = \frac{1}{a}\left(\sigma_{\tilde{x}} x(n) - \sigma_{\tilde{x}} d x(n-1) - a\eta\hat{w}(n-1)\right) \tag{10}$$

Considering the relationship between $x(n)$ and $a(n)$, the above expression can be rewritten as

$$\hat{w}(n) = \frac{\sigma_{\tilde{x}}}{a\sigma}\left(\log(a(n)) - d\log(a(n-1)) - (1-d)m\right) - \eta\hat{w}(n-1) \tag{11}$$

With the value of $\hat{w}(n)$, the item ν_2 can now be considered as a deterministic term. With the three terms Γ, ν_1 and ν_2, we can finally express the probability of $a(n+M)$ conditioned by $a(n), a(n-1), \ldots$, (i.e by $a(n)$ and $w(n)$) as

$$P(a(n+M)|a(n), a(n-1), \ldots) = \frac{1}{\sqrt{2\pi}\sigma_{\tilde{\Gamma}} a(n+M)} \exp\left(-\frac{\left(\log\left(\frac{a(n+M)}{\nu_1(a(n))\nu_2(w(n))}\right) - \mu_{\tilde{\Gamma}}\right)^2}{2\sigma_{\tilde{\Gamma}}^2}\right) \tag{12}$$

where we write ν_1 as $\nu_1(a(n))$ and ν_2 as $\nu_2(w(n))$ just to emphasize that the known terms $a(n)$ and $w(n)$ are included in them.

2.3. Prediction with the conditional probability

The predicted value of $\hat{a}(n+M)$ can be estimated by taking the expectation of the conditional probability (12)

$$\hat{a}(n+M) = \nu_1\nu_2 \exp\left(\frac{\sigma^2 a^2}{2\sigma_{\tilde{x}}^2}\left(1 + \sum_{i=1}^{M-1} d^{2i-2}(1+d)^2\right)\right) \tag{13}$$

with the standard deviation $\sigma_{\hat{a}(n+M)} = \nu_1\nu_2\sqrt{\exp(2\sigma_{\tilde{\Gamma}}^2) - \exp(\sigma_{\tilde{\Gamma}}^2)}$.

3. EXPERIMENT RESULTS

In this section, we illustrate the validity of our theoretical analysis. The filter coefficients of $H(z)$ were set to $a = 3.141 \times 10^{-4}$ and $d = 0.9994$. The parameters of the exponential module were set to $m = -0.3$ and $\sigma = 1.7$. These values correspond to typical analog prototypes obtained from several observation sites [4].

Firstly, we tested the consistency between the theoretical pdf (12) and that obtained by Monte-Carlo simulations. Supposing that $a(n) = 0.0521$, $w(n) = 0.5$, the values of $a(n+M)$ were independently generated 10000 times with $M = 1$, 4, 10, and 40 respectively. Histograms associated to these prediction steps are shown in Fig. 2. Theoretical distributions calculated by (12) are also depicted in the same figures. It can be clearly observed the theoretical results match the simulated data perfectly.

Secondly, the performance of the proposed algorithm was compared with other online prediction methods: (1) LMS algorithm: the LMS algorithm is a online implementation of the optimal linear filter with sequential input. However, LMS algorithm always suffers from an excess error due to its misadjustment [7]. (2) Log-LMS algorithm: it can be noticed that if the logarithm operation is applied on the model defined by (3), it becomes a linear prediction problem with respect to $x(n)$. Thus LMS algorithm can be used on this series and predicted $a(n+M)$ can be obtained by applying the reverse operation on $\hat{x}(n+M)$. The simulations were executed over fifty 10^6- sample independent sequences.The results are shown in the Fig. 3. The prediction with the mean of conditional pdf of $a(n+M)$ achieves the lowest mean square error and gives a performance bound for the other algorithms

4. REFERENCES

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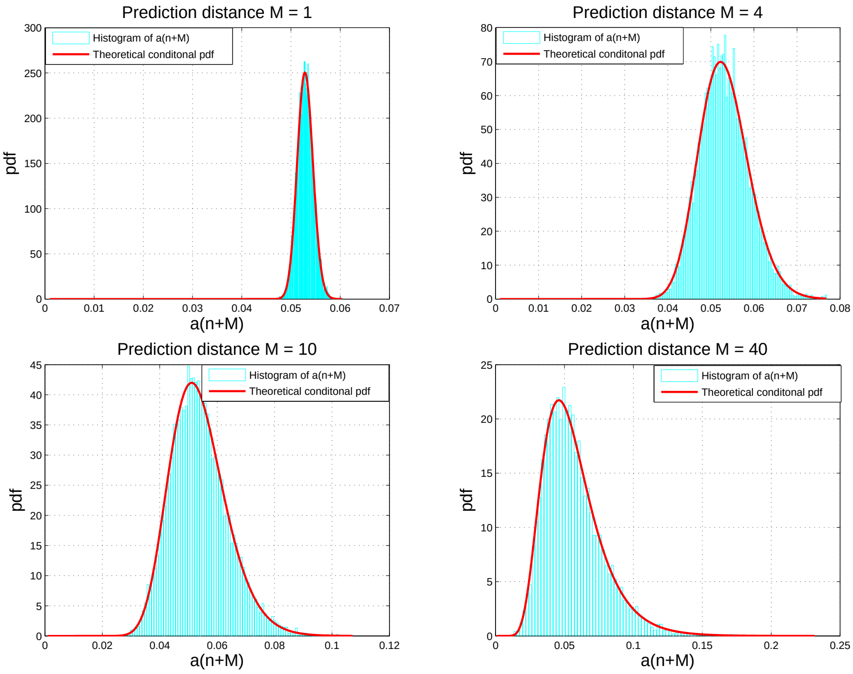

Fig. 2. Comparison between histograms of $a(n+M)$ and theoretical conditional distributions.

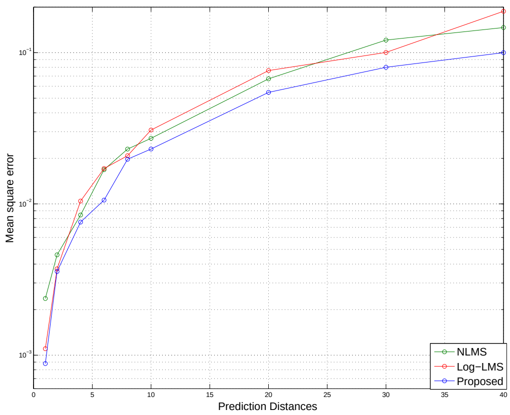

Fig. 3. Comparison with other online prediction algorithms

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